3.7 Example: Simply Supported Beam with Two Point Loads

3.7.1 Geometry, Loads and Bending Reinforcement

Consider the simply supported beam with point loads shown in Fig. 3-1 to be designed for a shear force V_d = 1'000 kN using a concrete grade C35 (f_{ck} = 35 MPa) and ordinary reinforcing steel S500 (f_{yk} = 500 MPa). The objective is to illustrate the application of the LoA's I and II to strut-and-tie and stress field models, for the design of a new structure. A very simple example is presented to clarify the application of the methodology. All checks and calculations will be based on *fib*'s Model Code for Concrete Structures 2010 (MC2010)[3-1] provisions.

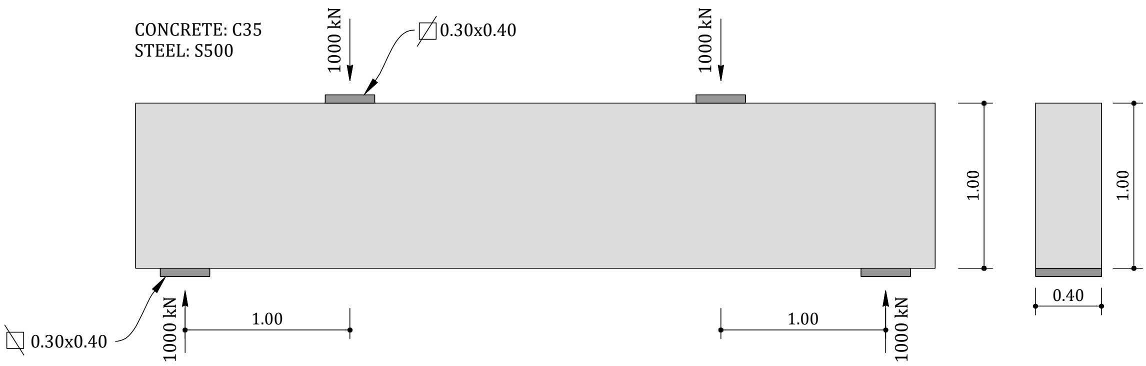


Fig. 3-1 Beam loads and geometry

Define the inner lever arm z, the bottom tension and the top compression at the mid span by applying the usual sectional method (to that aim, the concrete strength of the adjacent CCC node can be used to calculate the thickness of the compression chord). Calculate the longitudinal bottom reinforcement for the beam, provide minimum longitudinal web reinforcement and skin reinforcement (see Fig. 3-2).

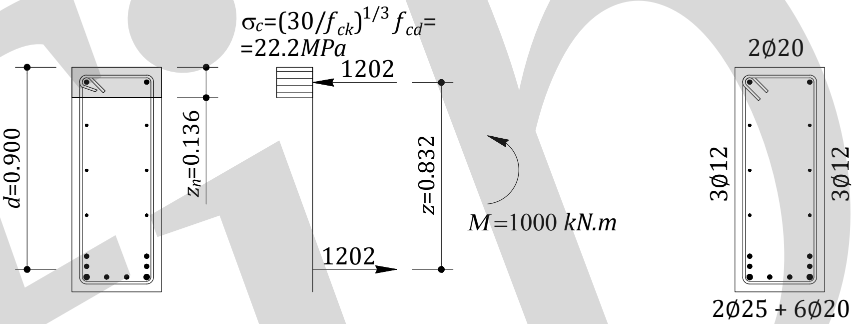


Fig. 3-2 Sectional method and calculation of the bending reinforcement

$$T = 1.202 \text{ MN} \Rightarrow A_{s,req} = 2'763 \text{ mm}^2 \Rightarrow 2\varnothing 25 + 6\varnothing 20; A_{s,prov} = 2'866 \text{ mm}^2 \quad (3\text{-}1)$$

3.7.2 Level of Approximation I

Step 1: Define a simple and safe-side strut-and-tie model. In this case a simple model consisting of a diagonal strut, from the point load to the support, is enough to calculate the main reinforcement and to check the nodal region. If necessary, a simple stress field model can be drafted to check any compression strength within the region. In most cases, however, the definition of the widths at the nodal region is sufficient to check compression and anchorage length (see Fig. 3-3).

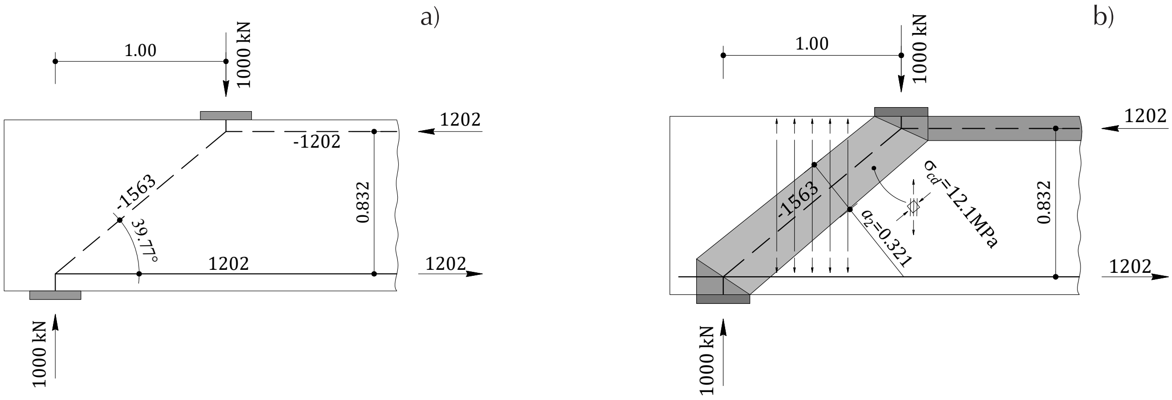


Fig. 3-3 a) Simple strut-and-tie model; b) Simple stress-field-model

Step 2: Since the point load is at a distance of the support $0.5<a/z<2.0$, part of the load must be transferred by stirrups to ensure an adequate service behaviour[3-2], [3-3], [3-4]. Stirrups' force may be calculated per $F_w = \frac{F}{3}(2\frac{a}{z}-1)$ and should be distributed over a width of approximately 0.75(a-0.3) ≈ 0.5m.

$$F_w = \frac{F}{3}\left(2\frac{a}{z}-1\right) = \frac{1'000}{3}\left(2\frac{1.0}{0.832}-1\right) = 468 \text{ kN} \tag{3-2}$$

$$f_w = 936 \text{ kN/m} \Rightarrow (A_s/s)_{req} = 2'152 \text{ mm}^2/\text{m} \Rightarrow \text{Stirrups } \varnothing 12//0.10; \quad (A_s/s)_{prov} = 2'262 \text{ mm}^2/\text{m} \tag{3-3}$$

Step 3: Check diagonal compression field. The diagonal strut develops in a region with vertical tension strains due to force at the stirrups. It is thus relevant to check the compression field with the reduced concrete compression strength for the considered concrete grade C35 (see Fig. 3-3, cracked concrete conditions). In the middle of the strut, it results: Eq. 3-4. Also check node stresses and anchorage length of the bars. Be consistent with the simple initial design model.

$$\sigma_c = 12.1 \text{ MPa} < \sigma_{c,Rd} = 0.55\left(\frac{30}{f_{ck}}\right)^{\frac{1}{3}} f_{cd} = 12.2 \text{ MPa} \tag{3-4}$$

In the simple model in Fig. 3-3 the bottom tension must be extended up to the support, thus the force to be anchored at nodal region shall be calculated accordingly. For the CCC node, the maximum compression is equal to the maximum allowable one (see Fig. 3-4b).

$$\sigma_c = 22.2 \text{ MPa} < \sigma_{c,Rd} = \left(\frac{30}{f_{ck}}\right)^{\frac{1}{3}} f_{cd} = 22.2 \text{ MPa} \tag{3-5}$$

For the CCT node, the bars are partially anchored over the support and the remaining force develops by bond behind the support. The design bond stresses ($f_{bd,p}$) in the support region may be increased due to the favourable effect of transverse pressure and outside support region bond stresses are given by f_{bd}. Furthermore, the contribution of the bends and hooks were considered by dividing the bond strength for straight bars by 0.7. According to MC2010: $f_{bd,p}$ = 3.94 MPa for the length over the plate and f_{bd} = 2.76 MPa for the outside length, providing the following bond forces.

$$F_b^{\varnothing 25} = n_{bars}\pi\,\varnothing \cdot l_{b1} \cdot f_{bd,p} = 2\cdot\pi\cdot 25\cdot 300\cdot 3.94\cdot 10^{-3} = 186\text{ kN}$$
$$F_b^{\varnothing 25} = n_{bars}\pi\,\varnothing \cdot l_{b2} \cdot f_{bd} = 2\cdot\pi\cdot 25\cdot 150\cdot 2.76\cdot 10^{-3} = 65\text{ kN}$$
$$F_b^{\varnothing 20} = n_{bars}\pi\,\varnothing \cdot l_{b1} \cdot f_{bd,p} = 6\cdot\pi\cdot 20\cdot 300\cdot 3.94\cdot 10^{-3} = 446\text{ kN} \qquad (3\text{-}6)$$
$$F_b^{\varnothing 20} = n_{bars}\pi\,\varnothing \cdot l_{b2} \cdot f_{bd} = 6\cdot\pi\cdot 20\cdot 150\cdot 2.76\cdot 10^{-3} = 156\text{ kN}$$
$$F_b^{total} = \frac{853}{0.7} = 1'219\text{ kN} < T_y = 435\cdot 2'866\cdot 10^{-3} = 1'246\text{ kN} > 1'202\text{ kN}$$

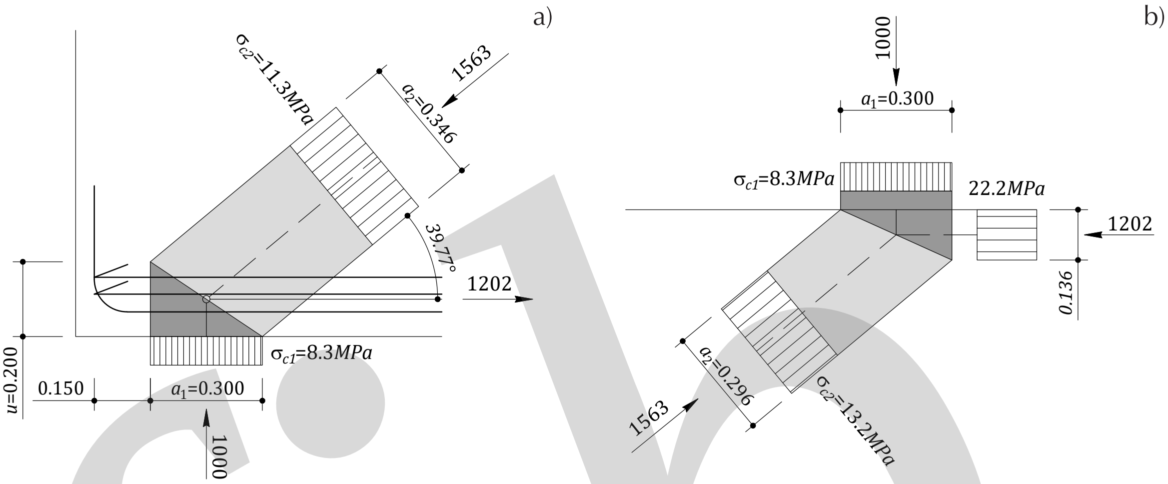


Fig. 3-4 a) CCT node; b) CCC node

Step 4: Full detailing for the beam, following the simple model, of transverse reinforcement and check of the nodal region. Note, that the calculated stirrups must be extended beyond the supports to provide confinement and to equilibrate the out-of-plane tension at node region. In the remaining region shear is zero and minimum shear reinforcement is provided.

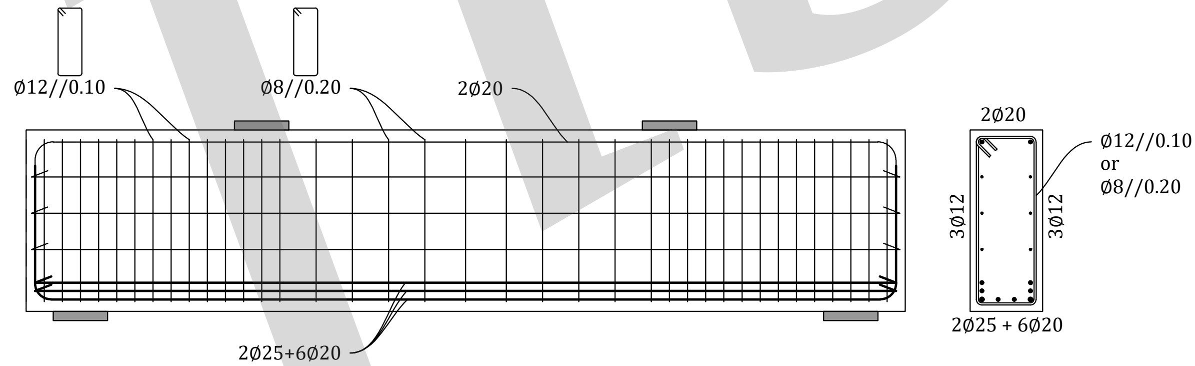


Fig. 3-5 Beam Detailing

3.7.3 Level of Approximation II

Step 1: Define the strut-and-tie model and include the necessary and relevant refinements within the model. In this case, include the required stirrups in the design model (see two possible models in Fig. 3-6a and Fig. 3-6b). Note that in this case, the required force to be anchored at the support is less than in LoA I (920 kN vs 1202 kN). For assessment of existing structures, one can also refine the model considering the horizontal reinforcement (see Fig. 3-6c or Fig. 3-6d), which leads to a further reduction of the bottom tension to be anchored at the support. Following these strut-and-tie models, the force to be anchored at the support is less than in LoA I, leading to a less critical situation than previously. A partial or full stress field model (see Fig. 3-7a or Fig. 3-7b) can be developed and all the necessary checks be performed. The diagonal compression stresses, the width of the distribution of stirrup and nodal widths and stresses, are clearly defined in the design model and all calculations and checks should be developed according to the design model.

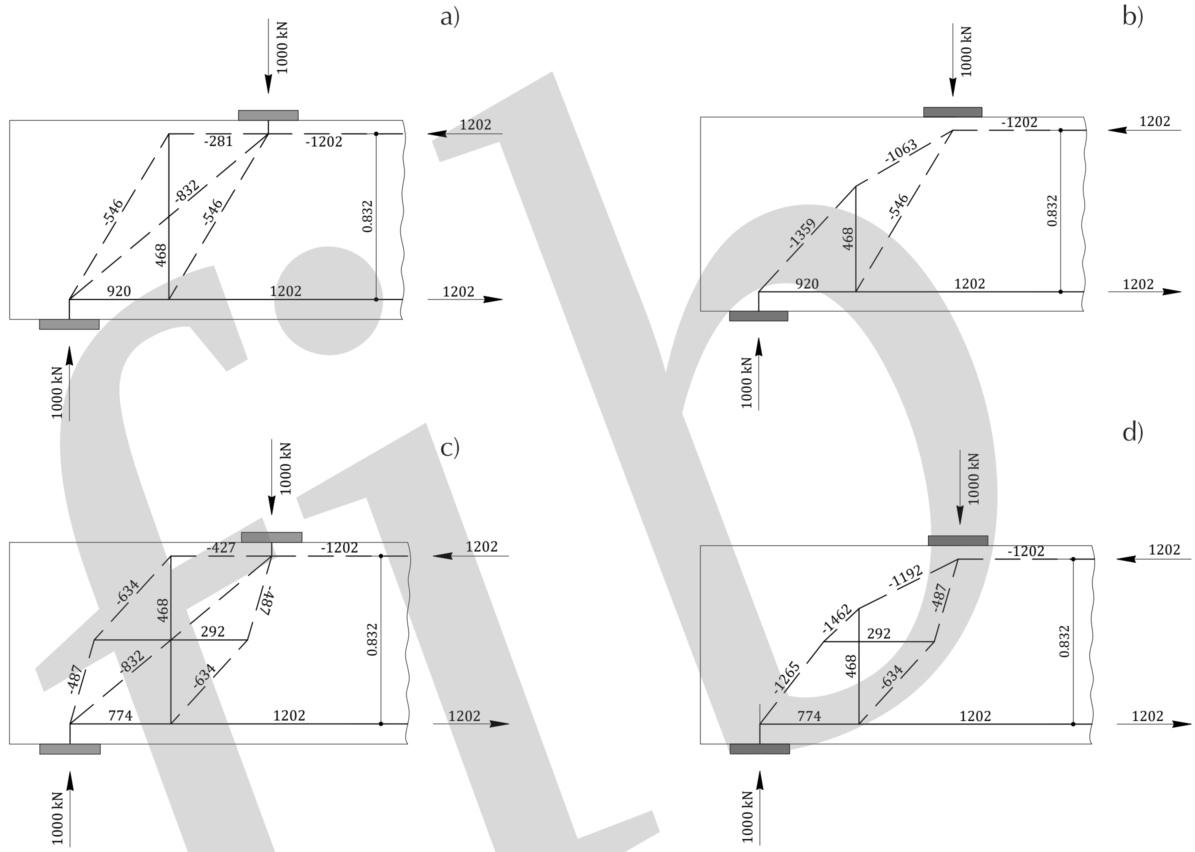
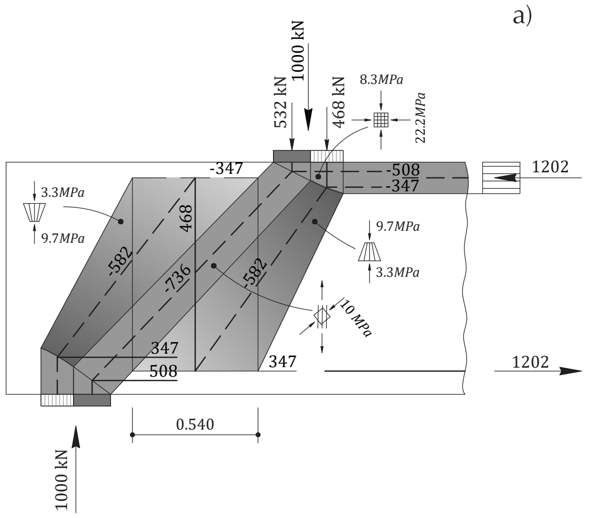


Fig. 3-6 a) Strut-and-tie model including load path carried by stirrups; b) Alternative strut-and-tie model; c) Strut-and-tie model including vertical and horizontal reinforcement; d) Alternative strut-and-tie model

Step 2: Check all compression strength within the model that may be critical (spreading of the strut leading to lower concrete stresses than LoA I). Follow the geometry and forces calculated for the stress field model. Check the bars anchorage length following the same procedure, as in LoA I.



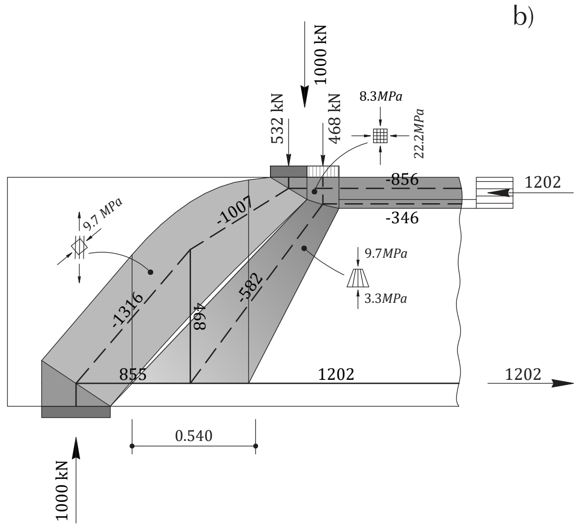


Fig. 3-7 a) Stress field model; b) Alternative stress field model

Diagonal strut stresses:

$$f_w = 468/0.54 = 867 \text{ kN/m} \Rightarrow (A_s/s)_{req} = 1'992 \text{ mm}^2/\text{m} \Rightarrow \text{Stirrups } \varnothing 12//0.10; \quad (A_s/s)_{prov} = 2'262 \text{ mm}^2/\text{m} \quad \text{3-7}$$

$$F_b^{total} = \frac{853}{0.7} = 1'219 \text{ kN} < Ty = 435 \cdot 2'866 \cdot 10^{-3} = 1'246 > 855 \text{ kM} \quad \text{3-8}$$

$$\sigma_c = 10 \text{ MPa} < \sigma_{c,Rd} = 0.55\left(\frac{30}{f_{ck}}\right)^{\frac{1}{3}} f_{cd} = 12.2 \text{ MPa} \quad \text{3-9}$$

$$\text{CCC node: } \sigma_c = 22.2 \text{ MPa} = \sigma_{c,Rd} = \left(\frac{30}{f_{ck}}\right)^{\frac{1}{3}} f_{cd} = 22.2 \text{ MPa} \quad \text{3-10}$$

$$\text{CCT node: } \sigma_c = 9.7 \text{ MPa} < \sigma_{c,Rd} = 0.75\left(\frac{30}{f_{ck}}\right)^{\frac{1}{3}} f_{cd} = 16.6 \text{ MPa} \quad \text{3-11}$$

Step 3: Full detailing for the beam following the design model.

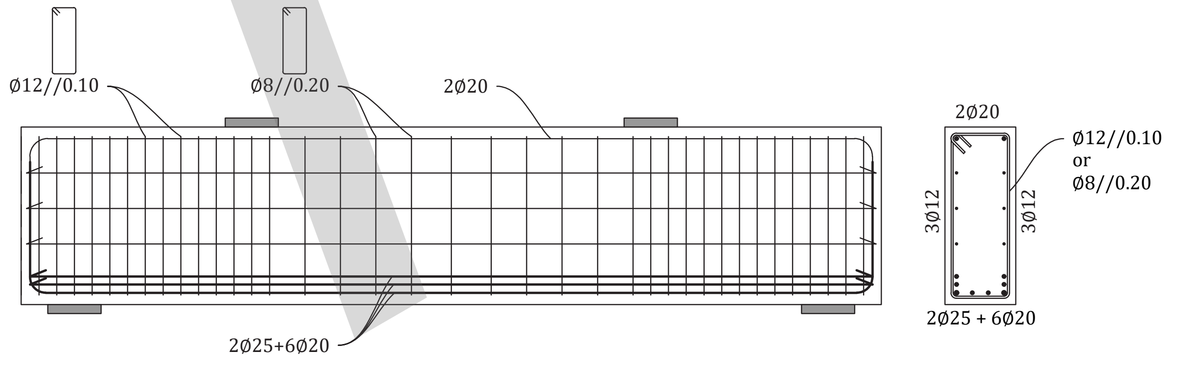


Fig. 3-8 Beam detailing

It should be noted that, despite the fact that the reinforcement layout remains the same as LoA I, the understanding of the member and its detailing is more consistent and refined (forces to be anchored over support, load spreading).

3.7.4 Level of Approximation III

Step 1: Apply a lower LoA to provide a first estimate of reinforcement. In this case the reinforcement layout presented in Fig. 3-8 will be analysed.

Step 2: A nonlinear finite element is developed. A simple triangle finite element for the concrete in considered without resistance in tension and a simplified elastic-plastic stress-strain law for compression. The reinforcement is simulated by a 1D element only with axial force and an elastic-plastic strain-stress law for tension and compression, neglecting any tension stiffening.

The output from the analysis is presented in Fig. 3-9a, with the average stress in the stirrups in the clear shear span depicted in Fig. 3-9b.

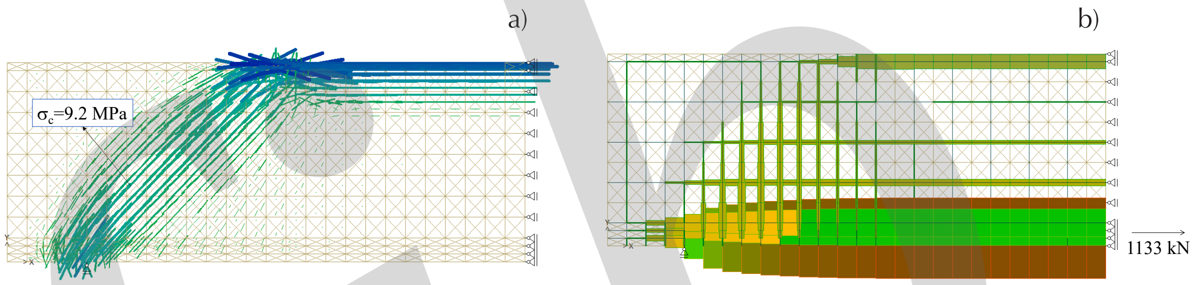


Fig. 3-9 Nonlinear finite element analysis results at ultimate load; a) compression stress fields; b) forces in the reinforcement

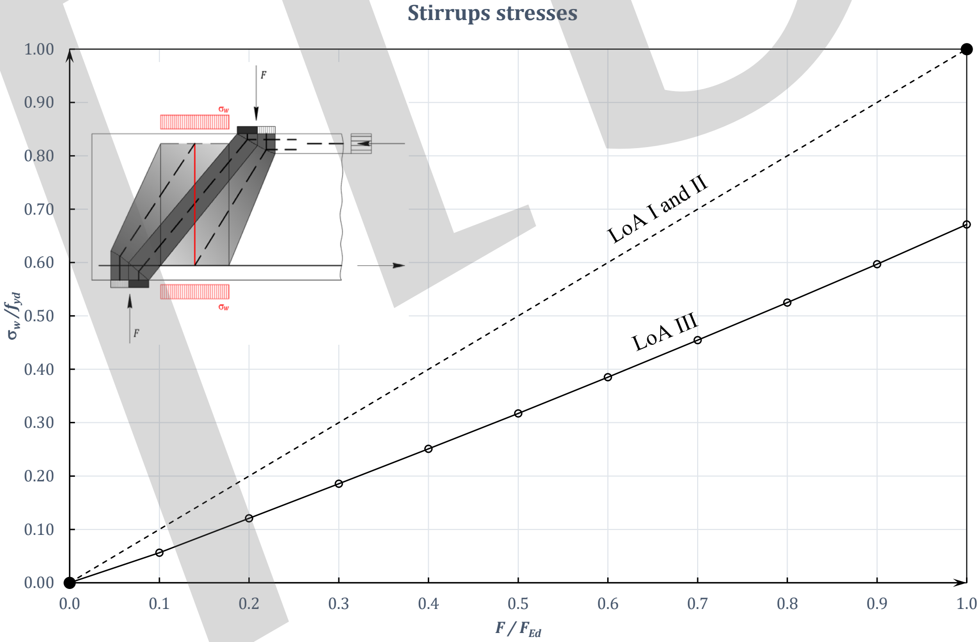


Fig. 3-10 Stresses in stirrups σ_w/f_{yd} vs applied force F under design force F_{Ed}.

With the non-linear analysis, it is possible to check the ultimate load and also some ductility aspects. The concrete compression softening due to the presence of transverse tensile strains is also considered to check any premature rupture and steel strains at peak load are lower than the ultimate steel strains. This shows that the semi-empirical Eq. 3-2 is conservative for this case, since in the LoA III analysis the stirrups steel stresses are approximately 300MPa for the design load.

Since no tension stiffening effect is considered it is not possible to draw any conclusion concerning the behaviour at service conditions.

3.7.6 Final considerations

Concerning the simple example presented some comments should be mentioned:

- LoA I is the safest and least time-consuming approach. The design model showed that the diagonal compression stresses and the anchorage length at the support are the critical issues. For the LoA II these topics were showed to be less critical.
- In LoA I the width in which the stirrups are distributed must follow the rules for this basic model. In the LoA II, the width may be obtained directly from the stress field model.
- The model refinement, developed for LoA II, provides valuable information, namely that the bottom tie force in the support is lower than in the LoA I. On the other hand, it also allowed the explicit calculation and check of the main compression stresses.
- The check of the CCT and CCC nodes is based in the simplified model (LoA I). In fact, the model considered in LoA II clearly showed that compression stresses in nodal region and the anchorage length are not as critical as determined in LoA I.
- Since LoA II is a redundant strut-and-tie model and compatibility is not explicitly computed in equilibrium models (such as LoA I and II), the stirrups force must be calculated following specific rules presented in technical bibliography. In the LoA III or higher, compatibility is explicitly considered and thus the force in the stirrups is obtained directly from the analysis.
- In LoA III, the obtained compression stresses and stirrup stresses at ultimate load are lower than the lowest LoA. No valuable information for service loads is provided in this level, so service behaviour must be checked indirectly as at the lowest levels.
- In the LoA IV, the most complex and, eventually, time-consuming analysis, is able to provide an explicit check of serviceability behaviour, as well as of deformation capacity.